DETERMINATION OF THERMOPHYSICAL CHARACTERISTICS FOR TWO-LAYER STRUCTURES FROM DATA OBTAINED IN NONSTEADY REGIME MEASUREMENTS

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Exact explicit functional relationships associating thermophysical characteristics of two-layer structures with parameters of nonsteady thermal action on a specimen are presented.

In many cases of actual practice there arises the need to determine the thermophysical characteristics of materials involving two-ply compositions (structural fragments) without destruction of their integrity. Using the approach outlined in [1-3] for this purpose, we can construct exact explicit relationships which relate the thermophysical characteristics of materials with the results obtained in the measurements of nonsteady surface temperatures and inward heat flows that change arbitrarily over time.

Let us examine the heat model of a two-ply specimen whose outside layer is subjected to the action of a heat flow $q(\tau)$, while the inside layer, because of the duration of the operation, may be regarded as a thermally "thick" layer (semibounded).

In the course of the experiment, let us record the temperature of the surface subjected to the action of the heat and to the incoming heat flow that arbitrarily changes with time. From the data obtained in the measurements of $T_1(\tau)$ and $q(\tau)$ we have to determine the thermophysical characteristics of the materials from which the two-ply specimen has been fabricated. A similar problem was examined in [4-6], where certain limitations were imposed on the conditions of the experiment, and the unknown parameters had no explicit expression in terms of the measurement results from the initial parameters, i.e., of the temperatures and heat flows.

Within the framework of a linear model of thermal conductivity, the mathematical formulation of the problem takes the form

$$\frac{\partial^2 T_1(\bar{r},\ \tau)}{\partial \bar{r}^2} = \frac{1}{a_1}\frac{\partial T_1(\bar{r},\ \tau)}{\partial \tau},\ 0 \leqslant \bar{r} \leqslant \delta_1; \tag{1}$$

$$\frac{\partial^2 T_2(r,\ \tau)}{\partial r^2} = \frac{1}{a_2}\frac{\partial T_2(r,\ \tau)}{\partial \tau},\ 0 \leqslant r < \infty; \tag{1'}$$

$$T_1(\delta_1,\ \tau) = T_1(\tau);\ \lambda_1 \frac{\partial T_1(\bar{r},\ \tau)}{\partial \bar{r}}\Big|_{\bar{r}=\delta_1} = q(\tau); \tag{2}$$

$$T_1(0,\ \tau) = T_2(0,\ \tau);\ -\lambda_1\frac{\partial T_1(\bar{r},\ \tau)}{\partial \bar{r}}\Big|_{\bar{r}=0} = \lambda_2\frac{\partial T_2(r,\ \tau)}{\partial r}\Big|_{r=0}; \tag{3}$$

$$T_2(r,\ \tau)\underset{r\to\infty}{=}0;\ T_1(\bar{r},\ 0) = T_2(r,\ 0) = 0; \tag{4}$$

$$a_1 = ?,\ \lambda_1 = ?,\ a_2 = ?,\ \lambda_2 = ? \tag{5}$$

On the basis of [7], the solution to system (1)-(4) in Laplace transform space can be presented in the form

$$T_1(\bar{r},\ s) = T_1(s)\frac{\mathrm{ch}\left(\sqrt{\frac{s}{a_1}}\bar{r}\right) + \varepsilon\,\mathrm{sh}\left(\sqrt{\frac{s}{a_1}}\bar{r}\right)}{\mathrm{ch}\left(\sqrt{\frac{s}{a_1}}\delta_1\right) + \varepsilon\,\mathrm{sh}\left(\sqrt{\frac{s}{a_1}}\delta_1\right)}, \tag{6}$$

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$$T_2(r,\ s) = T_1(s)\frac{\exp\left(-\sqrt{\frac{s}{a_2}}\,r\right)}{\mathrm{ch}\left(\sqrt{\frac{s}{a_1}}\,\delta_1\right)+\varepsilon\,\mathrm{sh}\left(\sqrt{\frac{s}{a_1}}\,\delta_1\right)},\quad \varepsilon^2=\frac{\lambda_2C_2}{\lambda_1C_1}. \tag{7}$$

With consideration of (2) we can write

$$\frac{q(s)}{\sqrt{s}\,T_1(s)} = \sqrt{\lambda_1C_1}\,\frac{\varepsilon+\mathrm{th}\left(\sqrt{\frac{s}{a_1}}\,\delta_1\right)}{1+\varepsilon\,\mathrm{th}\left(\sqrt{\frac{s}{a_1}}\,\delta_1\right)}. \tag{8}$$

Let us denote $q(s)s^{-1/2}T_1^{-1}(s) = \varphi$, so that

$$\mathrm{th}\left(\sqrt{\frac{s}{a_1}}\,\delta_1\right) = \frac{\varphi-\sqrt{\lambda_2C_2}}{\sqrt{\lambda_1C_1}-\varepsilon\varphi}. \tag{9}$$

Differentiating expression (9) with respect to s, we obtain

$$\frac{\delta_1}{2\sqrt{a_1s}}\left[1-\mathrm{th}^2\left(\sqrt{\frac{s}{a_1}}\,\delta_1\right)\right] = \frac{\varphi'(\sqrt{\lambda_1C_1}-\varepsilon\varphi)+(\varphi-\sqrt{\lambda_2C_2})\,\varepsilon\varphi'}{(\sqrt{\lambda_1C_1}-\varepsilon\varphi)^2} \tag{10}$$

and with consideration of (9)

$$\frac{\delta_1}{2\sqrt{a_1s}}\left[1-\frac{(\varphi-\sqrt{\lambda_2C_2})^2}{(\sqrt{\lambda_1C_1}-\varepsilon\varphi)^2}\right] = \varphi'\,\frac{\sqrt{\lambda_1C_1}-\varepsilon\sqrt{\lambda_2C_2}}{(\sqrt{\lambda_1C_1}-\varepsilon\varphi)^2}, \tag{11}$$

from which, after identical transformations, we obtain

$$\frac{\delta_1}{2\lambda_1\sqrt{s}}(\lambda_1C_1-\varphi^2) = \varphi', \tag{12}$$

or

$$\frac{\lambda_1}{\delta_1}\varphi' = \frac{\lambda_1C_1}{2\sqrt{s}} - \frac{\varphi^2}{2\sqrt{s}}. \tag{13}$$

After substitution of $\varphi = q(s)s^{-1/2}T_1^{-1}(s)$, we obtain

$$\frac{\lambda_1}{\delta_1}\left[q'(s)T_1(s)-q(s)T_1'(s)-\frac{1}{2s}q(s)T_1(s)\right] = \frac{\lambda_1C_1}{2}T_1^2(s)-\frac{q^2(s)}{2s}. \tag{14}$$

Thus, between the thermophysical characteristics of the heated surface in the case of a thermally "thick" inside wall we find the following functional relationship:

$$\frac{\lambda_1}{\delta_1}\psi_1(t) = \lambda_1C_1\psi_2(t)-\psi_3(t), \tag{15}$$

where

$$\psi_1(t) = \int_0^t (t-2\tau)\,q(\tau)\,T_1(t-\tau)\,d\tau - \frac{1}{2}\int_0^t T_1(t-\tau)\int_0^\tau q(\vartheta)\,d\vartheta d\tau,$$

$$\psi_2(t) = \frac{1}{2}\int_0^t T_1(\tau)\,T_1(t-\tau)\,d\tau,\quad \psi_3(t) = \frac{1}{2}\int_0^t q(t-\tau)\int_0^\tau q(\vartheta)\,d\vartheta d\tau.$$

It follows from an analysis of relationships (13)-(15) that the derived functional relationship between the thermophysical characteristics of the outer layer $q(\tau)$, $T_1(\tau)$ in explicit form does not contain the thermophysical characteristics of the inner layer, in which case the latter may be treated as a thermally "thick" wall. This circumstance apparently simplifies the problem of determining λ_1, a_1.

Indeed, based on relationships (15), in the presence of information with regard to $q(\tau)$, $T_1(\tau)$ in one or possibly two operations involving various laws $q(\tau)$ governing change over time for the determination of λ_1 and $\lambda_1 C_1$ we will have

$$\lambda_1 C_1 = \frac{\psi_{1i}\psi_{3j} - \psi_{1j}\psi_{3i}}{\psi_{1i}\psi_{2j} - \psi_{1j}\psi_{2i}}, \tag{16}$$

$$\frac{\lambda_1}{\delta_1} = \left(\frac{\psi_{1i}\psi_{2j} - \psi_{1j}\psi_{2i}}{\psi_{2i}\psi_{3j} - \psi_{2j}\psi_{3i}}\right)^{-1}, \tag{17}$$

where the subscripts i and j pertain to the various time intervals of a single operation or to some arbitrary time interval of two operations with differing $q(\tau)$. It should be noted that with "small" τ, when the outer layer has not yet been heated (it behaves as a thermally "thick" wall), the integral combination ψ_1 apparently differs insignificantly from zero, since the equality $(\lambda_1 C_1)^{1/2} T_1(s) = s^{-1/2} q(s)$ must be satisfied. Therefore, the parameter $\lambda_1 C_1$ may be determined on the basis of measurement data for $q(\tau)$ and $T_1(\tau)$ during the initial period of thermal action, i.e., $\lambda_1 C_1 = \psi_3/\psi_2$. Using the measurement data for $T_1(\tau)$ and $q(\tau)$ to calculate the integral ψ_1 and evaluating the difference of this integral from zero as a function of the time interval of the heat-treatment operation, we can easily determine the parameter λ_1/δ_1 by means of relationship (15), involving the use of the earlier found values for the parameter $\lambda_1 C_1$.

Let us note that the thermophysical characteristics of the outer layer can be obtained directly on the basis of (13)-(14) by calculation of the corresponding Laplace integrals of the experimental functions $T_1(\tau)$, $q(\tau)$. The required number of equations in this case for the determination of λ_1, $\lambda_1 C_1$ can be obtained by calculating the integrals contained in (13)-(14) for various values of the transform parameter s or by using the measurement data for $T_1(\tau)$, $q(\tau)$ in operations with different conditions of thermal action and by calculating the corresponding Laplace integrals for $T_1(\lambda)$, $q(\tau)$, $\tilde{T}_1(\tau)$, $\tilde{q}(\tau)$.

In addition to the above-cited relationships for the thermophysical characteristics of the outer layer, on the basis of relationship (12) we can obtain the calculated relationships in somewhat different form. It follows from (12) that

$$-\frac{\delta_1}{2\lambda_1}(\varphi^2)' = \left(\sqrt{s}\,\varphi' - \frac{\delta_1 C_1}{2}\right)', \tag{18}$$

from which, after a number of identical transformations, we obtain

$$\frac{\lambda_1}{\delta_1} = \frac{F_1(t)}{F_2(t)}, \tag{19}$$

where

$$F_1(t) = \int_0^t q(t-\tau) f_1(\tau)\,d\tau;\quad f_1(\tau) = \int_0^\tau (\tau - 2\vartheta) T_1(\tau - \vartheta)\tilde{q}(\vartheta)\,d\vartheta;$$

$$F_2(t) = \int_0^t (3\tau - 2t) T_1(t-\tau) f_2(\tau)\,d\tau + \frac{1}{2}\int_0^t T_1(t-\tau) f_1(\tau)\,d\tau;$$

$$f_2(\tau) = \frac{1}{\sqrt{\pi}}\int_0^\tau (\tau - 2\vartheta)\tilde{\tilde{q}}(\vartheta) T_1(\tau-\vartheta)\,d\vartheta - \int_0^\tau \tilde{q}(\vartheta) T_1(\tau - \vartheta)\,d\vartheta -$$

$$-\frac{q(0)}{\sqrt{\pi}}\int_0^\tau \frac{(\tau - 2\vartheta)}{\sqrt{\tau - \vartheta}} T_1(\vartheta)\,d\vartheta;$$

$$\tilde{q}(\vartheta) = \frac{1}{\sqrt{\pi}}\int_0^\vartheta \frac{q(\theta)}{\sqrt{\vartheta - \theta}}\,d\theta;\quad \tilde{\tilde{q}}(\vartheta) = \int_0^\vartheta \frac{q'(\theta)}{\sqrt{\vartheta - \theta}}\,d\theta.$$

After determination of the parameter λ_1/δ_1 on the basis of (19), the parameter $\lambda_1 C_1$ (or $a_1 = \lambda_1/C_1$) can be found by using formula (15):

$$\lambda_1 C_1 = \frac{F_1(t_i)\psi_1(t_j) + F_2(t_i)\psi_3(t_j)}{\psi_2(t_j)F_2(t_i)}, \tag{20}$$

or

$$\frac{a_1}{\delta_1^2} = \frac{F_2(t_i)}{F_1(t_i)}\left[\frac{\psi_1(t_j)}{\psi_2(t_j)} + \frac{F_2(t_i)}{F_1(t_i)}\,\frac{\psi_3(t_j)}{\psi_2(t_j)}\right]. \tag{21}$$

where t_i and t_j are the operational processing intervals.

If we take into consideration the information that we have with regard to the thermophysical characteristics of the outer layer, we can find the thermophysical characteristics of the inner layer on the basis of relationship (10), written in somewhat different form:

$$\frac{\delta_1}{2\sqrt{a_1 s}}\,\frac{1}{\mathrm{ch}^2\left(\sqrt{\frac{s}{a_1}}\,\delta_1\right)} = \frac{\varphi'(\lambda_1 C_1 - \lambda_2 C_2)}{\sqrt{\lambda_1 C_1}\,(\sqrt{\lambda_1 C_1} - \varepsilon\varphi)^2},$$

from which, having differentiated with respect to the parameter s and carrying out a series of identical transformations, we obtain

$$\lambda_1 C_1\left[\varphi'' + \frac{1}{2}\varphi' + \frac{\delta_1}{\lambda_1}\,\frac{\varphi\varphi'}{\sqrt{s}}\right] = (\lambda_2 C_2)^{1/2}\left[\varphi\varphi'' + \frac{1}{2}\varphi\varphi' s^{-1} + \frac{\delta_1 C_1}{\sqrt{s}}\varphi' - 2\varphi'^2\right]. \tag{22}$$

After substitution of $\varphi = s^{-1/2}q(s)T_1^{-1}(s)$ and returning to the originals for the determination of the parameter $\lambda_2 C_2$ we have

$$\sqrt{\lambda_2 C_2} = \frac{\lambda_1 C_1[\bar{\psi}_1(t) + \delta_1/\lambda_1\bar{\psi}_2(t)]}{\bar{\psi}_3(t) + \delta_1 C_1\bar{\psi}_4(t)}, \tag{23}$$

where

$$\bar{\psi}_1(t) = \int_0^t \bar{\varphi}(t-\tau)T_1(\tau)\,d\tau;\quad \bar{\psi}_2(t) = \int_0^t T_1(t-\tau)\int_0^\tau q(\vartheta)\bar{f}_1(\tau-\vartheta)\,d\vartheta d\tau;$$

$$\bar{\psi}_3(t) = \int_0^t \tilde{q}(\tau)\bar{\varphi}(t-\tau)\,d\tau - 2\int_0^t \bar{f}_1'(\tau)\bar{f}(t-\tau)\,d\tau;$$

$$\bar{\psi}_4(t) = \int_0^t T_1(t-\tau)\int_0^\tau \tilde{T}_1(\tau-\vartheta)\bar{f}_1'(\vartheta)\,d\vartheta d\tau;$$

$$\tilde{q}(\theta) = \frac{1}{\sqrt{\pi}}\int_0^\theta (\theta-\xi)^{-1/2}q(\xi)\,d\xi;\quad \tilde{\tilde{q}}(\theta) = \frac{1}{\sqrt{\pi}}\int_0^\theta (\theta-\xi)^{-1/2}q'(\xi)\,d\xi;$$

$$\tilde{T}_1(\vartheta) = \frac{1}{\sqrt{\pi}}\int_0^\vartheta (\vartheta-\theta)^{-1/2}T_1(\theta)\,d\theta;\quad \bar{f}_1(\vartheta) = \int_0^\vartheta (2\theta-\vartheta)\,T_1(\theta)\tilde{q}(\vartheta-\theta)\,d\theta;$$

$$\bar{f}_1'(\vartheta) = \int_0^\vartheta (\vartheta-\theta)\tilde{\tilde{q}}(\theta)T_1(\vartheta-\theta)\,d\theta - \int_0^\vartheta T_1(\vartheta-\theta)\tilde{q}(\theta)\,d\theta -$$

$$-\frac{1}{\sqrt{\pi}}q(0)\int_0^\vartheta (\vartheta-2\theta)(\vartheta-\theta)^{-1/2}T_1(\theta)\,d\theta;$$

$$\bar{\varphi}(\tau) = \int_0^\tau (2\tau-3\vartheta)\bar{f}_1'(\vartheta)T_1(\tau-\vartheta)\,d\vartheta - \frac{1}{2}\int_0^\tau T_1(\tau-\vartheta)\bar{f}_1(\vartheta)\,d\vartheta.$$

With these relationships we have exhausted the problem of determining the thermophysical characteristics of a two-ply composite material whose heat model assumes that the inside layer throughout the operation behaves similarly to a thermally "thick" wall.

Some interest has been shown in the possibility of using the above-derived functional relationships to determine the geometric parameters of a two-ply structure. For example, in dealing with a heat model with a thermally "thick" inside layer, we find that relationship (15) for certain thermophysical characteristics of the outer layer makes it possible to determine the thickness of that layer (on the basis of temperature measurements from the heated surface and from the intensity of the thermal effect on that surface) in accordance with the formula

$$\delta_1 = \frac{\lambda_1 \psi_1(\tau)}{\lambda_1 C_1 \psi_2(\tau) - \psi_3(\tau)}, \tag{24}$$

in which case we have no need for any information regarding the thermophysical characteristics of the inside layer, since this information in explicit form does not enter into the calculation relationship, but rather these data obviously make themselves felt in terms of the measured temperature values. Relationship (24), given here, exhibits a rather simple structure for practical utilization of the heat method of monitoring the film-coating thickness, etc., in those cases in which other methods, for a variety of reasons, cannot be utilized.

In order to test the validity of the derived relationships in the determination of the thermophysical characteristics of the outer layer when its thickness is known or to determine the thicknesses with known thermophysical characteristics, we made use of a numerical simulation experiment in which we used, as the initial "measurement" information, data from the solution of a series of direct thermal-conductivity problems involving diverse laws of variation over time for incoming heat flows with various thermophysical characteristics of the inner layer. Figure 1 shows the results of our estimates on the basis of data from a single operation with regard to the parameters λ_1 and $\lambda_1 C_1$ of the outer layer under conditions in which the action of the heat flow varies over time for various thermophysical characteristics of the inner layer [here the "true" value of the thermal conductivity of the outer layer in all of the experiments was assumed to be equal to 40 W/(m·K), and that of the parameter $\lambda_1 C_1$ was assumed to be equal to $0.144 \cdot 10^9$ $W \cdot J/(m^4 \cdot K^2)$, with the thickness of the outer layer amounting to 3 mm]. From the results shown in Fig. 1, it follows that in using "precise" initial data, we find that the calculated values for the parameters λ and $\lambda_1 C_1$ of the outer layer converge rather rapidly toward their true values with an increase in the duration of the operational intervals employed for the processing. Figure 2 shows the results from estimates of the thickness of the outer layer for known thermophysical characteristics in the case of different versions of the specified thermophysical characteristics of the inner layer and for various laws governing the change in time of the incoming heat flows. Just as in the case examined above, we note extremely rapid convergence of the calculated values for the parameter δ_1 to its true value (3 mm) with an increase in the duration of the operational time intervals employed for the processing.

The effect of the random errors in the initial information on the results of estimating the unknown parameters was studied through introduction into the "exact" original data, by means of a standard subprogram, of a random error with a uniform distribution. Table 1 shows the results from an estimate from the parameter λ_1 for the outer layer, involving the use of "measurement" data for $T_1(\tau)$ and $q(\tau)$, containing random errors with $\sigma_{T_1} = 0.01|T_1|$, $\sigma_q = 0.01|q|$. Here we processed the data from several operations with diverse heating conditions and different thermal conductivities for the inside layer; the volumetric heat capacity of the inside layer in all of the experiments amounted to $2.3 \cdot 10^6$ $J/(m^3 \cdot K)$. The columns under operation 1 show the results of estimating the parameter λ_1 where the flow of heat acts as a constant intensity over time, while the columns for operations 2 and 3 show the parameter estimates under the action of heat flows varying sinusoidally in time. From the results presented in Table 1 it follows that the presence of small random errors in the original information leads to slower convergence of the estimated parameter to its actual value as a function of the time intervals of the operations utilized in the processing. Nevertheless, for those operations used as an example we can assert completely acceptable accuracy in the estimates, which demonstrates the effectiveness of the proposed calculation algorithms.

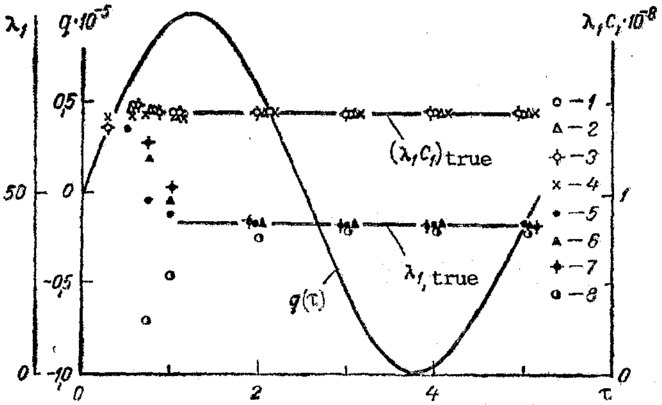

Fig. 1. Calculated values of the parameters $\lambda_1 C_1$ (1-4) and λ (5-8) for inside-layer thermal conductivities of 6, 25, 30, and 100 W/m·K, respectively. λ_1, W/m·K; $\lambda_1 C_1$, W·J/(m^4·K^2); q, W/m^2; τ, sec.

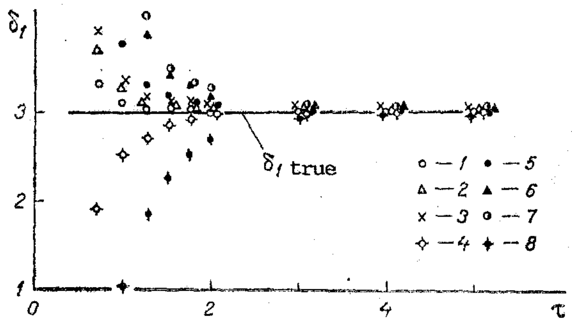

Fig. 2. Calculated values of outer-layer thickness: 1-4) δ_1 at constant q and 5-8) δ_1 with sinusoidal q in the case of an inside-layer thermal conductivity of 6, 25, 30, and 100 W/(m·K), respectively. δ_1, mm.

TABLE 1. Results from Estimates of the Parameter λ_1 in the Presence of Random Errors in the Initial Data; λ, W/(m·K)

τ, sec	Operation 1			Operation 2			Operation 3		
	λ_2								
	1	30	100	1	30	100	1	30	100
0,5	42,7	54,9	28,6	49,0	79,8	—	45,7	39,9	40,2
1,0	40,4	41,6	37,7	39,9	39,5	40,7	40,7	49,3	33,6
1,5	43,1	54,0	31,4	40,2	41,2	38,3	41,6	41,6	38,4
2,0	39,8	38,7	42,3	40,3	42,1	37,6	36,2	41,1	38,4
2,5	39,3	37,7	43,0	40,6	41,8	38,4	39,3	43,2	36,4
3,0	42,6	51,2	32,5	40,6	41,9	38,5	44,2	42,0	38,4
3,5	37,1	31,4	58,3	40,0	40,3	39,6	55,6	45,2	36,1
4,0	43,3	53,9	31,9	39,9	39,3	40,4	33,6	38,0	41,8
4,5	39,0	37,3	43,2	39,6	39,3	40,5	37,8	40,0	40,6

NOTATION

T, temperature; $\bar{r}$, r, space coordinates; τ, t, θ, ϑ, ξ, time; a_1, coefficient of thermal diffusivity for the outside layer; λ_1, coefficient of thermal conductivity for the outside layer; C_1, volumetric heat capacity of the outside layer; a_2, coefficient of thermal diffusivity for the inside layer; λ_2, coefficient of thermal conductivity for the inside layer; C_2, volumetric heat capacity of the inside layer; δ_1, thickness of the outside layer.

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CALCULATING CONVECTIVE HEAT EXCHANGE IN A HYPERSONIC VISCOUS SHOCK LAYER

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The streamlining of bodies with catalytic surfaces is investigated within the framework of a model of a hypersonic three-dimensional viscous shock layer.

With the motion of a body on an entry glide path the segment of the trajectory subjected to heat stresses lies in the region of nonequilibrium dissociation in which consideration must be given to a variety of physicochemical processes. The solution of such problems within the scope of total Navier–Stokes equations involves considerable difficulties, even when using the latest computer equipment, and a solution has been found only for axisymmetric flows. In order to carry out mass calculations it is expedient to employ simplified models and approximate relationships which permit estimates of the solution with retention of acceptable accuracy.

The present paper covers an investigation into the flows of heat to an indestructible blunt-body surface, and this study is based on the equations of a three-dimensional hypersonic viscous shock layer [1]. The equations describing the flow contain terms from the equations for the boundary layer and the nonviscous shock layer in hypersonic approximation. A model for a hypersonic or thin viscous shock layer was first proposed in [2] for two-dimensional flows. This model is an asymptotic form of the Navier–Stokes equations for large Mach and Reynolds numbers, as well as for the density ratios behind and in front of the shock wave, which is characteristic of the main portion of the glide trajectory.

The nonequilibrium chemical reactions and the multicomponent diffusion are taken into consideration in these equations. Thermal and pressure diffusion can be neglected. It is assumed that the internal degrees of freedom are excited in equal measure.

An absence of heat flow to the body is assumed in the boundary conditions at the surface of an impermeable body, and the effects of catalytic atom recombination at the wall are taken into consideration. The generalized Rankine–Hugoniot relationships are used as the boundary conditions at the shock wave, and these relationships allow us to take into consideration the effects of molecular transfer within the shock-wave zone.

The method of numerical solution is analogous to that described in [1]. Unlike that particular reference, provision is made in the equations for all of the nonsequential space measurement terms, thus allowing us more exactly to determine the flows of heat in regions of lower Reynolds numbers than was the case in [1].

Let us examine the flow in the vicinity of the critical point of a convex blunt body which is the point at which a plane perpendicular to the velocity vector of the approaching flow is tangent to the surface of the body. The equation for the surface of the body in this vicinity can be approximated to an accuracy of second-order terms by the equation of an elliptic paraboloid:

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